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Topological Indices On The Complement Of Mycielskian Graphs

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Let G be a simple connected graph with n vertices and m edges. The mycielski graph $\phi(G)$ of G contains G itself as an isomorphic subgraph, together with $n+1$ additional vertices: a vertex u_i corresponding to each vertex v_i of G and another vertex w. Each vertex u_i connected by an edge to w, so that these vertices form a subgraph in the form of a star $K_{1},_{n}$. In addition, for each edge (v_i, v_j) of G, the mycielski graph includes two edges (u_i, v_j) and (vi, u_j). In this paper we obtain First Zagreb index, First Zagreb Eccentricity index, Eccentric connectivity indices and polynomials and connective eccencticity index for the complement of Mycielski graphs. Furthermore, upper and lower bounds for the Harmonic index is also provided.

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I. Introduction

7cm Topological indices are numerical identities computed from the molecular graph of a chemical compound [14]. These indices are graph invariants which characterize the topology of that molecular graph. In addition to topological indices, a large number of graph polynomials are also introduced in the literature and most of them are found to be applicable in mathematical chemistry.

Let G be a simple connected graph with n vertices and m edges. By $V(G)$ and $E(G)$, we denote the vertex set and edge set of G respectively. For two vertices u and v of $V(G)$ their distance $d(u,v)$ is defined as the length of any shortest path connecting u and v in G. For a given vertex u of $V(G)$ its eccentricity $\varepsilon(u)$ is the largest distance between u and any other vertex v of G. Hence,

$$\varepsilon(u) = Max_{v \in V(G)} d(u,v).$$

The **First and Second Zagreb indices** [11] of a graph G, introduced by Gutman, are defined as

$$M_1(G) = \sum_{v_i \in V(G)} [d(v_i) + d(v_j)]$$

and

$$M_2(G) = \sum_{(v_i, v_j) \in E(G)} [d(v_i) d(v_j)]$$

respectively, where $d(v_i)$ is the degree of the vertex v_i in G.

The **Eccentric connectivity index** $\xi^c(G)$ of a graph G, proposed by Sharma, Goswami and Madan [13] is defined as the sum of the product of degree of a vertex and its eccentricity for all vertices of G. It is denoted by

$$\xi^c(G) = \sum_{v_i \in V(G)} d(v_i)\varepsilon(v_i)$$

, where $d(v_i)$ and $\varepsilon(v_i)$ are respectively the degree and eccentricity of vertex v_i.

The **Connective Eccentricity index** of a graph G [10] is a relatively new topological index defined as

$$C^{\xi}(G) = \sum_{v_i \in V(G)} \frac{d(v_i)}{\varepsilon(v_i)}$$

The **Eccentric connectivity polynomial** of a graph G is defined as [9] $\xi^c(G,x) = \sum d(v_i)x^{\varepsilon(v_i)}$, where $d(v_i)$ and $\varepsilon(v_i)$ are the degree and eccentricity of vertex v_i respectively, where value of x is greater than 1. The connection between the eccentric connectivity polynomial and the eccentric connectivity index is given by

$$\xi^c(G) = \xi^c(G,1)$$

where $\xi^c(G,1)$ is the first derivative of $\xi^c(G,x)$.

For a graph G the **First Zagreb Eccentricity index**[15] is defined as

$$ZE_1(G) = \sum_{v_i \in V(G)} \varepsilon(v_i)^2.$$

The **Harmonic index** $H(G)$ for a graph G is a variant of Randi c' connectivity index. It is defined as $H(G) = \sum_{v_i, v_j \in E(G)} [\frac{2}{d(v_i)+d(v_j)}]$ [7, 8, 16].

The **Mycielski graph** $\phi(G)$ of G, put forward by Mycielski [12] and well studied by authors in [1, 3, 4], contains G itself as an isomorphic subgraph and n+1 additional vertices : vertices $u_1, u_2, ...u_n$ corresponding to vertices $v_1, v_2, ...v_n$ of G and a new vertex w of degree n. Each vertex u_i connectedto w and for each edge (v_i, v_j) of G, the mycielski graph contains two edges (u_i, v_j) and (vi, u_j). In $\phi(G)$, there are 4 types of edges $-$ edges joining vertices v_i and v_j, u_i and v_j, v_i and u_j and u_i and w. Moreover we have the following lemmas.

$1.$ If u be a nontrivial graph of order n and size m, and let $\phi(G)$ be its mycielski graph. Then for each $i = 1,2,..n$, we have $d_{\phi(G)}(v_i) = 2d_G(v_i)$, $d_{\phi(G)}(u_i) = d_G(v_i)+1$ and $d_{\phi(G)}(w) = n$ [2].

$2.$ Let G be a triangle- free graph with vertex set $v = v_1, v_2, ...v_n$. Let $\phi(G)$ be the mycielski graph. Then we have the following results [6] .

1. $d_{\phi(G)}(v_i, v_j) = \begin{cases} d_G(v_i, v_j), & if\ d_G(v_i, v_j) \le 3; \\ 4, & if\ d_G(v_i, v_j) \ge 4. \end{cases}$

2. $d_{\phi(G)}(u_i,v_j) = \begin{cases} 2, & if\ i=j; \\ d_G(v_i,v_j), & if\ d_G(v_i,v_j) \le 3, i \ne j; \\ 3, & if\ d_G(v_i,v_j) \ge 4, i \ne j. \end{cases}$

3. $d_{\phi(G)}(v_i,w) = d_{\phi(G)}(u_i,u_j) = 2, d_{\phi(G)}(u_i,w) = 1. ie., d_{\phi(G)}(v_i,v_j) \le d_G(v_i,v_j)$

The Complement of a graph G, denoted by $\overline{G}$ is the graph with the same vertices as that of G and two vertices are adjacent in $\overline{G}$ if they are not adjacent in G In order to determine the various topological indices of the complement of the Mycielski graph $\overline{\phi}(G)$, we need the following observations.

1. $d_{\overline{\phi}(G)}(v_i) = 2n - 2d_G(v_i)$, $d_{\overline{\phi}(G)}(u_i) = 2n - 1 - d_G(v_i)$ and $d_{\overline{\phi}(G)}(w) = n$.

2. $\varepsilon_{\overline{\phi}(G)}(v_i) = \varepsilon_{\overline{\phi}(G)}(u_i) = \varepsilon_{\overline{\phi}(G)}(w) = 2$.

Various mathematical properties of Mycielsky graph has been studied extensively by various authors. For instance Chvatal in [3], discussed about the minimality property of this graph. A lower bound on the chromatic number of Mycielski graph was reported by the authors in [1] and in 2004 Collins and Tysdal discussed about dependent edges in mycielski graphs[4]. In [5] authors invetigated Zagreb coindices of mycielski graphs and explicit formulae are derived in terms of Zagreb indices and coindices of their parent graphs.

In this paper various topological indices of complement of Mycielski graphs are obtained. Throughout this paper we consider compliment of the Mycielski graph with diameter exactly 2.

II. First Zagreb index and First Zagreb Eccentricity index

In this section, two topological indices such as First Zagreb inded and First Zagreb Eccentricity index of the complement of a Mycielski graph are computed.

Theorem 1 *Let G be an n vertex graph of size m and let $\overline{\phi}(G)$ be the complement of the Mycielskian ϕ of G. Then the First Zagreb index of $\overline{\phi}(G)$ is given by*

$$M_1(\overline{\phi}(G)) = 5M_1(G) + n(8n^2 - 3n + 1) + 4m(6n+1)$$

. *Proof.*

$$\begin{aligned} M_1(\overline{\phi}(G)) &= \sum_{(v_i \in V(\overline{\phi}(G))} d(v_i)^2 \\ &+ \sum_{u_i \in V(\overline{\phi}(G))} d(u_i)^2 + d(w)^2. \\ &= \sum_{v_i \in V(G)} (2n - 2d(v_i))^2 \\ &+ \sum_{v_i \in V(G)} (2n - 1 - d(v_i))^2 + n^2. \\ &= \sum_{v_i \in V(G)} [(4n^2 - 8n(d(v_i)) + 4(d(v_i)^2) \\ &+ (2n-1)^2 + 2(2n-1)d(v_i) + (d(v_i))^2] \\ &= 4n^2 - 16mn + 4M_1(G) + n(4n^2 + 1 - 4n) \\ &+ 4m(2n-1) + M_1(G) \\ &= 5M_1(G) + n(8n^2 - 3n + 1) + 4m(6n+1) \end{aligned}$$

Theorem 2 *The First Zagreb eccentricity index of* $\overline{\phi}(G)$ *is given by* $ZE_1(\overline{\phi}(G)) = 4(2n+1).$

Proof. The First Zagreb eccentricity index of a graph G is $ZE_1(G) = \sum_{v_i \in V(G)} \varepsilon^2(v_i).$ Hence

$$\begin{aligned} ZE_1(\overline{\phi}(G)) &= \sum_{v_i \in V(\overline{\phi}(G))} \varepsilon^2(v_i) + \sum_{u_i \in V(\overline{\phi}(G))} \varepsilon^2(u_i) \\ &+ \sum_{w \in V(\phi(G))} \varepsilon^2(w) \\ &= 4n + 4n + 4 \\ &= 4(2n+1) \end{aligned}$$

III. Eccentric connectivity index, Eccentric connectivity polynomial and connective eccentricity index

Three eccentricity based topological indices are considered and their values are derived for the complement of the mycielsky graph G .

Theorem 3 *Let* G *be an* n *vertex graph of size* m *and let* $\overline{\phi}(G)$ *be the complement of the Mycielskian* ϕ *of* G *. Then, the Eccentric connectivity index of* $\overline{\phi}(G)$ *is given by*
$\xi^c(\overline{\phi}(G)) = 4(2n^2 - 3m)$

Proof.

$$\begin{aligned} \xi^c(\overline{\phi}(G)) &= \sum_{v_i \in V(\overline{\phi}(G))} d(v_i)\varepsilon(v_i) + \sum_{u_i \in V(\overline{\phi}(G))} d(u_i)\varepsilon(u_i) \\ &+ \sum_{w \in V(\phi(G))} d(w)\varepsilon(w) \\ &= \sum_{v_i \in V(G)} 2(n - d(v_i))2 \\ &+ \sum_{u_i \in V(G)} (2n - 1 - d(v_i))2 + 2n \\ &= 8n^2 - 12m \\ &= 4(2n^2 - 3m) \end{aligned}$$

Corollary 4 *The Eccentric connectivity polynomial of* $\overline{\phi}(G)$ *is given by*
$\xi^c(\overline{\phi}(G,x)) = x^2(4n^2 - 6m).$

Proof.

$$\begin{aligned} \xi^c(\overline{\phi}(G,x)) &= \sum_{v_i \in V(G)} 2(n - d(v_i))x^2 \\ &+ \sum_{u_i \in V(G)} (2n - 1 - d(v_i))x^2 + nx^2 \\ &= x^2(4n^2 - 6m). \end{aligned}$$

Theorem 5 *The Connective eccentricity index of complement of the Mycielski graph is* $C^{\overline{\xi}}(G) = 2n^2 - 3m.$

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Proof. The connective eccentricity index of a graph G is given by, $C^{\xi}(G) = \sum_{v_i \in V(G)} \frac{d(v_i)}{\varepsilon(v_i)}$

Therefore

$$
\begin{aligned}
C^{\bar{\xi}}(G) &= \sum_{v_i \in V(\bar{\phi}(G))} \frac{d(v_i)}{\varepsilon(v_i)} \\
&+ \sum_{u_i \in V(\bar{\phi}(G))} \frac{d(u_i)}{\varepsilon(u_i)} \\
&+ \sum_{w \in V(\bar{\phi}(G))} \frac{d(w)}{\varepsilon(w)} \\
&= \sum_{v_i \in V(G)} \frac{2(n-d(v_i))}{2} \\
&+ \sum_{v_i \in V(G)} \frac{2n-1-d(v_i)}{2} + \frac{n}{2} \\
&= n^2 - 2m + \frac{n(2n-1)}{2} - m + \frac{n}{2} \\
&= 2n^2 - 3m.
\end{aligned}
$$

IV. Bounds of Harmonic index on Mycielsky graphs

In this section, we provide upper and lower bounds for the Harmonic index of the mycielsky graphs.

Theorem 6 *Let G be a (n,m) graph with maximum degree and minimum degree Δ and δ respectively. The mycielsky graph of G is denoted by $\phi(G)$. Then*

$$\frac{1}{2}H(G) + \left\{\frac{4m}{3\Delta+1} + \frac{2n}{1+\Delta+n}\right\} \le H(\phi(G)) \le \frac{1}{2}H(G) + \left\{\frac{4m}{3\delta+1} + \frac{2n}{1+\delta+n}\right\}$$

Proof. By the definition of Harmonic index, we have

$$H(G) = \sum_{(v_i, v_j \in E(G))} \frac{2}{d(v_i) + d(v_j)}$$

Hence by considering the various types of edges in $\phi(G)$, one can write,

$$
\begin{aligned}
H(\phi(G)) &= \sum_{(v_i, v_j) \in E(\phi(G))} \frac{2}{d(v_i) + d(v_j)} \\
&+ \sum_{(v_i, u_j) \in E(\phi(G))} \frac{2}{d(v_i) + d(u_j)} \\
&+ \sum_{(u_i, w) \in E(\phi(G))} \frac{2}{d(u_i) + w}
\end{aligned}
$$

Now consider the term

$$\sum_{(v_i,u_j)\in E(\phi(G))}\frac{2}{d(v_i)+d(u_j)}=\sum_{(v_i,v_j)\in E(G)}\frac{2}{2d(v_i)+(1+d(v_j))}$$
$$\leq\sum_{(v_i,v_j)\in E(G)}\frac{2}{2\delta+(1+\delta)}$$
$$=\frac{4m}{3\delta+1}.$$

Similarly, we can see that

$$\frac{4m}{3\Delta+1}\leq\sum_{(v_i,u_j)\in E(\phi(G))}\frac{2}{d(v_i)+d(u_j)}$$

It is also obvious that,

$$\sum_{(u_i,w)\in E(\phi(G))}\frac{2}{d(u_i)+w}=\sum_{v_i\in V(G)}\frac{2}{(1+d(v_i))+n}$$
$$\leq\frac{2n}{1+\delta+n}$$

Therefore,

$$\frac{2n}{1+\Delta+n}\leq\sum_{(u_i,w)\in E(\phi(G))}\frac{2}{d(u_i)+w}\leq\frac{2n}{1+\delta+n}$$

Finally we have,

$$\sum_{(v_i,v_j)\in E(\phi(G))}\frac{2}{d(v_i)+d(v_j)}=\sum_{(v_i,v_j)\in E(G)}\frac{2}{2(d(v_i)+d(v_j))}$$
$$=\frac{1}{2}H(G)$$

Hence the proof.

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